Positive Announcements

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ABSTRACT

Arbitrary public announcement logic reasons about how the knowledge of a set of agents changes in response to (possibly arbitrary) public announcements of true epistemic formulas. We consider a variant of arbitrary public announcement logic called *arbitrary positive announcement logic*, which restricts arbitrary public announcements to positive formulas. Positive formulas prohibit statements about the ignorance of agents, and as a result have useful properties that make their effects more predictable and well-behaved.

We present preliminary results in this logic, showing that it is more expressive than public announcement logic for multiple agents, that it is not at least as expressive as arbitrary public announcement logic, that the model-checking problem is PSPACE-complete and that there is a sound and complete axiomatisation. We conjecture that, unlike arbitrary public announcement logic, the satisfiability problem for this logic is decidable.

Keywords: Arbitrary public announcement logic, dynamic epistemic logic, epistemic protocol synthesis, modal logic, multi-agent systems, refinements, positive formulas.

CR Classifications: F.4.1 [Theory of Computation] Mathematical Logic and Formal Languages – Mathematical Logic.

1. INTRODUCTION

Dynamic epistemic logics consider how knowledge changes as a result of the communication of information in a multi-agent system. Public announcement logic in particular is concerned with reasoning about the effects of public announcements of true epistemic statements. The public nature of the communication means that every agent receives the communication, every agent knows that every agent receives the communication, every agent knows that every agent knows that every agent receives the communication, and so on. The effect of publicly announcing a statement is often that the statement becomes common knowledge amongst the agents, that is, every agent knows that the statement is true, every agent knows that every agent knows that the statement is true, and so on. If a public announcement of a statement results in common knowledge of the statement we say that it was a successful public announcement. However as public announcements can include statements about the ignorance of agents, there are cases where a statement that is true before announcement becomes false after announcement, and such statements cannot be successful public announcements. For example, if the Moore sentence "It is raining and you don't know that it is raining" is true, then publicly announcing this fact will cause it to become false, so it cannot become common knowledge (one cannot *know* something that is false).

Public announcement logic (PAL) [9, 13] extends epistemic logic with operators for reasoning about the effects of specific public announcements. The formula $[\psi]\varphi$ means that "φ is true after the truthful announcement of ψ". Arbitrary public announcement logic ($APAL$) [3] augments this further with operators for reasoning about arbitrary public announcements. The formula $\Box\varphi$ means that "φ is true after the truthful announcement of any epistemic formula". Quantifying over the communication of information as in $APAL$ has applications to epistemic protocol synthesis problems, where we wish to achieve epistemic goals by communicating information to agents, but we don't know of a specific protocol that will achieve the goal and may not even know if such a protocol exists. In principle these problems can be solved by specifying problems as formulas in the logic and applying model-checking or satisfiability procedures. However in the case of $APAL$, while there is a PSPACE-complete model-checking procedure [1] and a recursively enumerable, sound and complete axiomatisation [3], the satisfiability problem is undecidable with multiple agents [8].

We consider instead a variant of $APAL$ called *arbitrary positive announcement logic* ($APAL^+$). We restrict arbitrary public announcements to positive formulas, so the formula $\boxplus\varphi$ means that "φ is true after the truthful announcement of any *positive* formula". Positive formulas consist only of positive knowledge statements, such as "it is known that" rather than "it is not known that", prohibiting statements such as the Moore sentence above.

From an applications perspective this is a natural restriction. Many practical knowledge bases are expressed in formalisms with restrictions similar to positive formulas, as this allows for efficient reasoning and querying. For example, the very large biomedical ontology SNOMED CT is expressed in a subset of the description logic $\mathcal{EL}^{++}$, which permits existential quantifiers but not universal quantifiers, allowing

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reasoning and querying tasks to be performed in polynomial time [2]. Synthesising epistemic protocols with similar restrictions would ensure that the protocol can be executed on such knowledge bases. Network protocol synthesis problems, such as the sequence-transmission problem, have also been reasoned about using formalisms where agents send and receive messages and acknowledgements that are represented as positive formulas [11, 12]. An agent acknowledges receiving a message by sending the message that they "know" that the previous message was sent. In this setting it is not possible to send a denial of receiving a message, as that would presuppose knowing that the message was sent.

In contrast to public announcements, positive announcements have a number of properties that make their results easier to reason about. All true positive announcements are successful, the truth of a positive announcement is preserved under any subsequent public announcement, and repeating a positive announcement has no effect. Unlike public announcements, not every finite sequence of positive announcements can be expressed as a single positive announcement, so there is a distinction between quantifying over individual positive announcements and sequences of positive announcements. We conjecture that the restriction to positive formulas is enough to make the satisfiability problem decidable.

In this paper we present some preliminary results in $APAL^+$. In Section 2 we recall the required technical preliminaries, such as definitions and results from epistemic logic, PAL and $APAL$. In Section 3 we introduce the syntax and semantics of $APAL^+$, and present semantics results about the properties of positive announcements and the arbitrary positive announcement operators. In Section 4 we show that the model-checking problem is PSPACE-complete. In Section 5 we compare the expressivity of $APAL^+$ to PAL and $APAL$, showing that it is equally expressive to PAL for a single agent, and strictly more expressive for multiple agents, and that it is not at least as expressive as $APAL$ for multiple agents. In Section 6 we provide an axiomatisation and show that it is sound and complete. Finally in Section 7 we outline on-going work and open questions.

2. TECHNICAL PRELIMINARIES

We recall definitions and technical results from epistemic logic, public announcement logic [9, 13] and arbitrary public announcement logic [3].

Let A be a finite set of agents and let P be a countable set of propositional atoms.

Definition 2.1. *An* epistemic model $M = (S, \sim, V)$ *consists of a* domain S*, which is a non-empty set of states (or possible worlds), a set of* accessibility relations $\sim$*, indexed by agents* $a \in A$*, where* $\sim_a \subseteq S \times S$ *is an equivalence relation on states (a relation that is reflexive, transitive and symmetric), and a* valuation $V : P \to \mathcal{P}(S)$*, which is a function from states to sets of propositional atoms.*

The class of all epistemic models *is called* $\mathcal{S}5$*. A* pointed epistemic model $M_s = ((S, \sim, V), s)$ *consists of an epistemic model* M *along with a designated state* $s \in S$ *(the real world).*

Given two states $s, t \in S$, we write $s \sim_a t$ to denote that $(s, t) \in \sim_a$. We write $[s]_a$ to denote the a-equivalence class of s, which is the set of states $[s]_a = \{t \in S \mid s \sim_a t\}$. As we will often be required to discuss several models at once, we will use the convention that $M_s = ((S, \sim, V), s)$, $M'_{s'} = ((S', \sim', V'), s')$, $M^{\gamma}_{s^{\gamma}} = ((S^{\gamma}, \sim^{\gamma}, V^{\gamma}), s^{\gamma})$, etc.

An epistemic model is an abstract model of the knowledge of a set of agents, defined over a set of *possible worlds*. Atomic propositions are true or false in a world depending on whether the world appears in the valuation for the atomic proposition. The accessibility relation for an agent defines which possible worlds are *indistinguishable* to the agent. An agent is said to *know* a statement is true about a possible world, such as the real world, if the statement is true in every world that is indistinguishable to the agent from the possible world.

Definition 2.2. *The* language of epistemic logic $\mathcal{L}_{el}$ *is defined inductively as:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi$$

where $p \in P$ *and* $a \in A$*.*

We use all of the standard abbreviations for propositional logic, in addition to the abbreviation $L_a\varphi ::= \neg K_a \neg\varphi$. The formula $K_a\varphi$ is read as "agent a knows that φ is true", and $L_a\varphi$ is read as "agent a considers it possible that φ is true".

We now define the semantics of epistemic formulas on epistemic models.

Definition 2.3. *Let* $M_s = ((S, \sim, V), s) \in \mathcal{S}5$ *be an epistemic model. The interpretation of* $\varphi \in \mathcal{L}_{el}$ *in the* epistemic logic *S5* *is defined inductively as:*

$$\begin{array}{lll} M_s \models p & \text{iff} & s \in V(p) \\ M_s \models \neg\varphi & \text{iff} & M_s \not\models \varphi \\ M_s \models \varphi \wedge \psi & \text{iff} & M_s \models \varphi \text{ and } M_s \models \psi \\ M_s \models K_a\varphi & \text{iff} & \text{for every } t \sim_a s : M_t \models \varphi \end{array}$$

If $M_s \models \varphi$ we say that M_s *satisfies* φ. If $M_s \models \varphi$ for every $s \in S$ we say that M satisfies φ and we simply write $M \models \varphi$. If $M \models \varphi$ for every $M \in \mathcal{S}5$ we say that φ is *valid* in $\mathcal{S}5$ and we simply write $\models \varphi$. If there exists $M_s \in \mathcal{S}5$ such that $M_s \models \varphi$ then we say that φ is *satisfiable*. Given an epistemic model $M = (S, \sim, V) \in \mathcal{S}5$ and a formula φ we define $[\![\varphi]\!]_M ::= \{s \in S \mid M_s \models \varphi\}$ to be the set of states where φ is satisfied in M.

Example 2.4. *Let* $M_s = ((S, \sim, V), s) \in \mathcal{S}5$ *be an epistemic model where* $S = \{s, t, u, v\}$*,* $s \sim_a t$*,* $u \sim_a v$*,* $s \sim_b u$ *and* $t \sim_b v$*,* $V(p) = \{s, t\}$ *and* $V(q) = \{u, v\}$*. This model is represented in Figure 1. The atomic proposition* p *is true in every possible world that agent* a *cannot distinguish from the real world,* s*, so we can say that "agent* a *knows that* p *is true in the real world", written as* $M_s \models K_a p$*. However agent* b *cannot distinguish between the real world and a possible world where* p *is false, and so* $M_s \models \neg K_b p$*. We also have that* $M_t \models \neg K_b p$*, and so* $M_s \models K_a \neg K_b p$*.*

Figure 1: An example of an epistemic model.

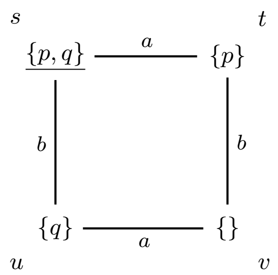
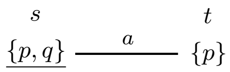


Figure 2: The result of publicly announcing $K_a p$ in the epistemic model of Figure 1.



Definition 2.5. *The* language of public announcement logic $\mathcal{L}_{pal}$ *is defined inductively as:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid [\varphi]\varphi$$

where $p \in P$ *and* $a \in A$.

We use all of the standard abbreviations for epistemic logic, in addition to the abbreviation $\langle\varphi\rangle\psi ::= \neg[\varphi]\neg\psi$.

Definition 2.6. *Let* $M = (S, \sim, V) \in \mathcal{S5}$ *be an epistemic model and* $T \subseteq S$. *We define the* restriction of M to T *as* $M|T = (S|T, \sim|T, V|T)$ *where:*

$$\begin{aligned} S|T &= T \\ \sim_a|T &= \sim_a \cap (T \times T) \\ V|T(p) &= V(p) \cap T \end{aligned}$$

Let $\mathcal{L}$ *be a language with an interpretation on* M, *and let* $\varphi \in \mathcal{L}$. *We define the* restriction of M to φ *as* $M|\varphi = M|[\![\varphi]\!]_M$.

A model restriction to a formula φ restricts the possible worlds to those where φ was true before the model restriction. This is the basis of public announcements.

Definition 2.7. *Let* $M = (S, \sim, V) \in \mathcal{S5}$ *be an epistemic model. The interpretation of* $\varphi \in \mathcal{L}_{pal}$ *in the* public announcement logic *PAL is the same as its interpretation in the epistemic logic S5 given in Definition 2.3 along with the additional inductive case:*

$$M_s \vDash [\varphi]\psi \quad \textit{iff} \quad \textit{if } M_s \vDash \varphi \textit{ then } M_s|\varphi \vDash \psi$$

Example 2.8. *Consider the epistemic model* M_s *from Example 2.4. Then* $[\![K_a p]\!]_M = \{s, t\}$, *and so* $M_s|K_a p$ *is the model restriction of* M_s *to the states* $\{s, t\}$, *depicted in Figure 2. In the resulting model restriction we have that* $M_s|K_a p \vDash K_b p$, *and so in the original model we therefore have* $M_s \vDash \langle K_a p\rangle K_b p$. *That is, agent b knows that p is true after it is publicly announced that agent a knows that p is true.*

Lemma 2.9. *PAL is expressively equivalent to epistemic logic (for single or multiple agents).*

This result is shown by Plaza [13].

Definition 2.10. *The* language of arbitrary public announcement logic $\mathcal{L}_{apal}$ *is defined inductively as:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid [\varphi]\varphi \mid \Box\varphi$$

where $p \in P$ *and* $a \in A$.

We use all of the standard abbreviations for public announcement logic, in addition to the abbreviation $\Diamond\varphi ::= \neg\Box\neg\varphi$.

Definition 2.11. *Let* $M = (S, \sim, V) \in \mathcal{S5}$ *be an epistemic model. The interpretation of* $\varphi \in \mathcal{L}_{apal}$ *in the* arbitrary public announcement logic *APAL is the same as its interpretation in the public announcement logic PAL given in Definition 2.7 along with the additional inductive case:*

$$M_s \vDash \Box\varphi \quad \textit{iff} \quad \textit{for every } \psi \in \mathcal{L}_{el} : M_s \vDash [\psi]\varphi$$

Example 2.12. *Consider the epistemic model* M_s *from Example 2.4. In Example 2.8 we showed that* $M_s \vDash \langle K_a p\rangle K_b p$ *so it follows that* $M_s \vDash \Diamond K_b p$.

Proposition 2.13. *APAL is expressively equivalent to PAL for a single agent.*

Proposition 2.14. *APAL is strictly more expressive than PAL for multiple agents.*

These results are shown by Balbiani, et al. [3].

Definition 2.15. *Let* $M = (S, \sim, V) \in \mathcal{S5}$ *and* $M' = (S', \sim', V') \in \mathcal{S5}$ *be epistemic models. A non-empty relation* $\mathfrak{R} \subseteq S \times S'$ *is a* bisimulation *if and only if for every* $(s, s') \in \mathfrak{R}$, $p \in P$, *and* $a \in A$ *the conditions* **atoms-**p, **forth-**a *and* **back-**a *hold.*

atoms-p $s \in V(p)$ *if and only if* $s' \in V'(p)$.

forth-a *For every* $t \sim_a s$ *there exists* $t' \sim'_a s'$ *such that* $(t, t') \in \mathfrak{R}$.

back-a *For every* $t' \sim'_a s'$ *there exists* $t \sim_a s$ *such that* $(t, t') \in \mathfrak{R}$.

If $(s, s') \in \mathfrak{R}$ *then we call* M_s *and* $M'_{s'}$ bisimilar *and write* $M_s \simeq M'_{s'}$.

Lemma 2.16. *The relation* $\simeq$ *is an equivalence relation on epistemic models.*

Lemma 2.17. *Let* $M_s, M'_{s'} \in \mathcal{S5}$ *be epistemic models such that* $M_s \simeq M'_{s'}$ *and let* $\varphi \in \mathcal{L}_{el}$ *be an epistemic formula. Then* $M_s \vDash \varphi$ *if and only if* $M'_{s'} \vDash \varphi$.

LEMMA 2.18. *Let $M, M' \in \mathcal{S5}$ be image-finite epistemic models (each state has finitely many accessible states) and let $\mathfrak{R} \subseteq S \times S'$ be a relation such that $(s, s') \in \mathfrak{R}$ if and only if for every $\varphi \in \mathcal{L}_{el}$: $M_s \models \varphi$ if and only if $M'_{s'} \models \varphi$. Then $\mathfrak{R}$ is a bisimulation.*

These are well-known results.

DEFINITION 2.19. *Let $M, M' \in \mathcal{S5}$ be epistemic models and let $Q \subseteq P$ be a set of propositional atoms. A non-empty relation $\mathfrak{R} \subseteq S \times S'$ is a Q-*bisimulation *if and only if for every $(s, s') \in \mathfrak{R}$, $q \in Q$, and $a \in A$, the conditions* **atoms-**q, **forth-**a *and* **back-**a *hold. If $(s, s') \in \mathfrak{R}$ then we call M_s and $M'_{s'}$ Q-*bisimilar *and write $M_s \simeq_Q M'_{s'}$.*

We note that analogous results to Lemma 2.16, Lemma 2.17 and Lemma 2.18 apply to Q-bisimulations when we restrict the language of epistemic formulas to propositional atoms in Q.

DEFINITION 2.20. *Let $M, M' \in \mathcal{S5}$ be epistemic models. A non-empty relation $\mathfrak{R} \subseteq S \times S'$ is a* refinement *if and only if for every $(s, s') \in \mathfrak{R}$, $p \in P$, and $a \in A$, the conditions* **atoms-**p *and* **back-**a *hold. If $(s, s') \in \mathfrak{R}$ then we call $M'_{s'}$ a* refinement *of M_s and call M_s a* simulation *of $M'_{s'}$. We write $M'_{s'} \preceq M_s$ or equivalently $M_s \succeq M'_{s'}$.*

DEFINITION 2.21. *The* language of positive formulas $\mathcal{L}^+_{el}$ *is defined inductively as:*

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_a\varphi$$

where $p \in P$ and $a \in A$.

LEMMA 2.22. *All true positive formulas are preserved under public announcements.*

LEMMA 2.23. *All true positive formulas are successful as public announcements.*

COROLLARY 2.24. *All true positive formulas are idempotent as public announcements.*

These results were shown by van Ditmarsch and Kooi [7].

LEMMA 2.25. *The relation $\preceq$ is a preorder on epistemic models.*

LEMMA 2.26. *Let $M_s, M'_{s'} \in \mathcal{S5}$ be epistemic models such that $M_s \succeq M'_{s'}$ and let $\varphi \in \mathcal{L}^+_{el}$ be a positive formula. If $M_s \models \varphi$ then $M'_{s'} \models \varphi$.*

These results are shown by van Ditmarsch, French and Pinchinat [6].

LEMMA 2.27. *Let $M, M' \in \mathcal{S5}$ be image-finite epistemic models and let $\mathfrak{R} \subseteq S \times S'$ be a relation such that $(s, s') \in \mathfrak{R}$ if and only if for every $\varphi \in \mathcal{L}^+_{el}$: if $M_s \models \varphi$ then $M'_{s'} \models \varphi$. Then $\mathfrak{R}$ is a refinement.*

This result follows from similar reasoning to Lemma 2.18.

3. SYNTAX AND SEMANTICS

In this section we give the syntax and semantics of $APAL^+$, and we provide some semantic results about the properties of positive announcements and the arbitrary positive announcement operators.

DEFINITION 3.1. *The* language of arbitrary positive announcement logic $\mathcal{L}_{apal^+}$ *is defined inductively as:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid [\varphi]\varphi \mid \boxplus\varphi$$

where $p \in P$ and $a \in A$.

We use all of the standard abbreviations for public announcement logic, in addition to the abbreviation $\oplus\varphi ::= \neg\boxplus\neg\varphi$.

DEFINITION 3.2. *Let $M = (S, \sim, V) \in \mathcal{S5}$ be an epistemic model. The interpretation of $\varphi \in \mathcal{L}_{apal^+}$ in the* positive announcement logic $APAL^+$ *is the same as its interpretation in the public announcement logic PAL given in Definition 2.7 along with the additional inductive case:*

$$M_s \models \boxplus\varphi \quad \text{iff} \quad \text{for every } \psi \in \mathcal{L}^+_{el} : M_s \models [\psi]\varphi$$

EXAMPLE 3.3. *In Example 2.12 we showed that $M_s \models \Diamond K_b p$, as $M_s \models \langle K_a p\rangle K_b p$. As $K_a p$ is a positive formula it follows also that $M_s \models \oplus K_b p$.*

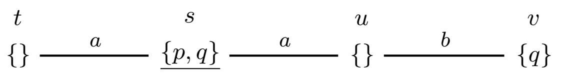
An important observation is the partial correspondence between the results of positive announcements and model restrictions that are closed under refinements, a notion that we will define now.

DEFINITION 3.4. *Let $M = (S, \sim, V) \in \mathcal{S5}$ be an epistemic model and let $T \subseteq S$ be a set of states. We say that* T is closed under refinements in M *if and only if for every $s, t \in S$ such that $M_s \succeq M_t$: if $s \in T$ then $t \in T$. We say that the model restriction $M|T$* is closed under refinements *if and only if T is closed under refinements in M.*

LEMMA 3.5. *The result of any positive announcement is closed under refinements.*

PROOF. Let $M = (S, \sim, V) \in \mathcal{S5}$ be an epistemic model and let $\varphi \in \mathcal{L}^+_{el}$. Suppose that $s, t \in S$ such that $s \in [\![\varphi]\!]_M$ and $M_s \succeq M_t$. Then $M_s \models \varphi$. As $M_s \succeq M_t$ and $\varphi \in \mathcal{L}^+_{el}$ then by Lemma 2.26 we have $M_t \models \varphi$. So $t \in [\![\varphi]\!]_M$ and therefore $M|\varphi$ is closed under refinements. $\square$

Figure 3: An example of an epistemic model with public announcements that do not correspond to any positive announcement.



Lemma 3.6. *On finite models, a model restriction that is closed under refinements is the result of a positive announcement.*

Proof. Let $M = (S, \sim, V) \in \mathcal{S5}$ be an epistemic model and let $T \subseteq S$ be a set of states such that $M|T$ is closed under refinements. Then for every $s \in T$ and $t \in S \setminus T$ we have that $s \not\succeq t$ and as M is finite then by Lemma 2.27 there exists $\varphi_{s,t} \in \mathcal{L}^+_{el}$ such that $M_s \models \varphi_{s,t}$ but $M_t \not\models \varphi_{s,t}$. Let $\varphi = \bigvee_{s \in T} \bigwedge_{t \in S \setminus T} \varphi_{s,t}$. Then $\varphi \in \mathcal{L}^+_{el}$; for every $s \in T$: $M_s \models \varphi$; and for every $t \in S \setminus T$: $M_t \not\models \varphi$. So $[\![\varphi]\!]_M = T$ and therefore $M|T$ is the result of a positive announcement. $\square$

We use these results to give an example of where public announcements and positive announcements differ.

Example 3.7. *Consider $M_s = ((S, \sim, V), s) \in \mathcal{S5}$, where $S = \{s, t, u, v\}$, $s \sim_a t \sim_a u$, $u \sim_b v$, $V(p) = \{s\}$ and $V(q) = \{s, v\}$. We note that $M_u \succeq M_t$, and so by Lemma 3.5 any positive announcement that preserves u must also preserve t. Therefore we have that $M_s \models \boxplus(K_a L_b q \rightarrow K_a p)$, that is, any positive announcement that results in agent a knowing that agent b considers q possible will also result in agent a knowing that p is true. However we note that $M_s \models \langle L_b q\rangle(K_a L_b q \wedge \neg K_a p)$ and so $M_s \not\models \Box(K_a L_b q \rightarrow K_a p)$.*

Similar reasoning can be used to show that, in contrast to public announcements, a sequence of positive announcements cannot generally be expressed as a single positive announcement.

Proposition 3.8. *Arbitrary positive announcements are not composable in $\mathcal{S5}$, i.e. it is not the case that $\mathcal{S5} \models \diamondplus\diamondplus\varphi \rightarrow \diamondplus\varphi$ for all $\varphi \in \mathcal{L}_{apal^+}$.*

Proof. We construct a counter-example.

Let $M = (S, \sim, V)$ where $S = \{s, t, u, v, w, t', u', v'\}$, $s \sim_a t \sim_a t'$, $u \sim_a v \sim_a w$, $u' \sim_a v'$, $t \sim_b u$, $t' \sim_b u'$, $V(p) = \{s, t, u, v, t', u', v'\}$, and $V(q) = \{t, v, t'v'\}$.

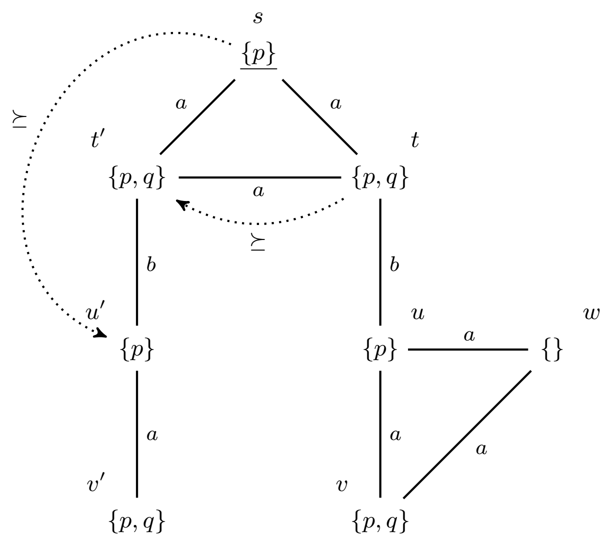
The model M is represented in Figure 4.

We claim that $M_s \models \diamondplus\diamondplus(L_a q \wedge K_a(K_b q \vee K_b \neg q))$ but $M_s \not\models \diamondplus(L_a q \wedge K_a(K_b q \vee K_b \neg q))$.

We note that $M_s|K_a p|(K_b q \vee K_b \neg q) \models L_a q \wedge K_a(K_b q \vee K_b \neg q)$ and so $M_s \models \diamondplus\diamondplus(L_a q \wedge K_a(K_b q \vee K_b \neg q))$.

Let $\mathfrak{R} = \{(x, x) \mid x \in S\} \cup \{(t, t'), (u, u'), (v, v'), (t', v'), (s, u')\}$. We note that $\mathfrak{R}$ is a refinement.

Figure 4: Counterexample for the composability of positive announcements.



As $M_t \succeq M_{t'}$ then by Lemma 3.5 any positive announcement that preserves t will also preserve t' so any positive announcement that results in $L_a q$ will preserve t'. As $M_s \succeq M_{u'}$ then by Lemma 3.5 any positive announcement will preserve u', so any positive announcement that results in $L_a q$ will also result in $\neg K_a(K_b q \vee K_b \neg q)$. Therefore $M_s \models \boxplus(L_a q \rightarrow \neg K_a(K_b q \vee K_b \neg q))$ and so $M_s \not\models \diamondplus(L_a q \wedge K_a(K_b q \vee K_b \neg q))$. $\square$

This lack of composability is interesting because it raises questions about what is possible with finite sequences of positive announcements of varying lengths. We could introduce to the language operators of the form $\diamondplus^n$ for $n \in \mathbb{N}$, where $\diamondplus^n\varphi$ means that "φ is true after the truthful announcement of some sequence of positive formulas of length n". This would be definable through abbreviations in $\mathcal{L}_{apal^+}$. We could also introduce an operator $\diamondplus^*$ for finite sequences of positive announcements of unbounded length, where $\diamondplus^*\varphi$ means that "φ is true after the truthful announcement of some finite sequence of positive formulas". We currently do not know whether this adds expressivity or complexity to the logic.

We continue, with the results that positive announcements have the Church-Rosser and McKinsey properties. The proofs by Balbiani, et al. [3] of these properties for public announcements are not valid for positive announcements, so we present alternative reasoning.

Lemma 3.9. *Let $Q \subseteq P$ be a set of propositional atoms and let $M_s \in \mathcal{S5}$ such that $M_s \simeq_Q M_s|\{s\}$. Then for every $\varphi \in \mathcal{L}_{apal^+}$ consisting only of atoms from Q: $M_s \models \varphi$ if and only if $M_s|\{s\} \models \varphi$.*

Proof. By induction over subformulas of φ, we show for every subformula ψ of φ and every state $t \in S$ reachable from s that $M_t \vDash \psi$ if and only if $M_s|\{s\} \vDash \psi$. All inductive cases are straightforward except perhaps for the case for $\boxplus\psi$. We note for every set of states $T \subseteq S$ and every state $t \in T$ that $M_t|T \simeq_Q M_t|\{t\}$ so by transitivity we have that $M_t|T \simeq_Q M_s|\{s\}$. So for every positive formula $\varphi \in \mathcal{L}_{el}^+$ if $M_t \vDash \boxplus\psi$ then $M_t|\varphi \simeq_Q M_s|\{s\}$ so $M_s|\{s\} \vDash \varphi$ and therefore $M_s|\{s\} \vDash \boxplus\varphi$. □

PROPOSITION 3.10. *Arbitrary positive announcements have the Church-Rosser property in $\mathcal{S5}$, i.e. $\mathcal{S5} \vDash \diamondplus\boxplus\varphi \rightarrow \boxplus\diamondplus\varphi$ for all $\varphi \in \mathcal{L}_{apal^+}$.*

PROOF. Let $M_s = ((S, \sim, V), s) \in \mathcal{S5}$ be an epistemic model and let $\varphi \in \mathcal{L}_{apal^+}$ be a formula such that $M_s \vDash \diamondplus\boxplus\varphi$. Let $Q \subseteq P$ be the set of propositions that appear in φ. We define the formula $\ell_s^Q = \bigwedge_{s \in V(q)}^{q \in Q}(q) \wedge \bigwedge_{s \notin V(q)}^{q \in Q}(\neg q)$ and note that $\ell_s^Q \in \mathcal{L}_{el}^+$. If $M_s \vDash \diamondplus\boxplus\varphi$ then there is a positive formula $\psi \in \mathcal{L}_{el}^+$ such that after announcing ψ every positive announcement will cause φ to be true. Therefore $M_s|\psi \vDash \boxplus\varphi$ and so $M_s|\psi|\ell_s^Q \vDash \varphi$. However, regardless of the announcement ψ we have that $M_s|\psi|\ell_s^Q \simeq_Q M_s|\{s\}$, so by Lemma 3.9 it follows that $M_s|\{s\} \vDash \varphi$. Now suppose that $\psi \in \mathcal{L}_{el}^+$ is a positive formula such that $M_s \vDash \psi$. Then $M_s|\psi|\ell_s^Q \simeq_Q M_s|\{s\}$. From above we have that $M_s|\{s\} \vDash \varphi$ so by Lemma 3.9 we have that $M_s|\psi|\ell_s^Q \vDash \varphi$, and so $M_s \vDash \boxplus\diamondplus\varphi$ as required. □

PROPOSITION 3.11. *Arbitrary positive announcements have the McKinsey property in $\mathcal{S5}$, i.e. $\mathcal{S5} \vDash \boxplus\diamondplus\varphi \rightarrow \diamondplus\boxplus\varphi$ for all $\varphi \in \mathcal{L}_{apal^+}$.*

PROOF. This proof is very similar to the proof for the Church-Rosser property. Let $M_s = ((S, \sim, V), s) \in \mathcal{S5}$ be an epistemic model and let $\varphi \in \mathcal{L}_{apal^+}$ be a formula such that $M_s \vDash \boxplus\diamondplus\varphi$. Let $Q \subseteq P$ be the set of propositions that appear in φ. Recall the definition of ℓ_s^Q from the proof of Proposition 3.10. Then we have that $M_s|\ell_s^Q \vDash \diamondplus\varphi$. We can see for any positive formula $\psi \in \mathcal{L}_{el}^+$ that $M_s|\ell_s^Q|\psi \simeq_Q M_s|\{s\}$, so by Lemma 3.9 it follows that $M_s|\{s\} \vDash \varphi$. By this reasoning we must also have that $M_s|\ell_s^Q \vDash \boxplus\varphi$ and so $M_s \vDash \diamondplus\boxplus\varphi$ as required. □

Finally we note that $APAL^+$, like $APAL$, is not compact.

A (possibly infinite) set of formulas is *satisfiable* in a logic if and only if there exists an epistemic model $M_s \in \mathcal{S5}$ that satisfies every formula from the set. A set of formulas is *finitely satisfiable* in a logic if and only if every finite subset of the set of formulas is satisfiable. A logic is *compact* if and only if every finitely satisfiable set of formulas is satisfiable in the logic.

PROPOSITION 3.12. *$APAL^+$ is not compact.*

This follows from the same reasoning used by Balbiani, et al. [3] to show that $APAL$ is not compact. Specifically, a set of formulas $\{[\psi](K_a p \rightarrow K_b K_a p) \mid \psi \in \mathcal{L}_{el}^+\} \cup \{\neg\boxplus(K_a p \rightarrow K_b K_a p)\}$ is demonstrated which is finitely satisfiable but not satisfiable under the semantics of $APAL^+$.

4. MODEL-CHECKING

In this section we show that the model-checking problem for $APAL^+$ is PSPACE-complete. The *model-checking problem* for $APAL^+$ is to determine for a given formula $\varphi \in \mathcal{L}_{apal^+}$ and epistemic model $M_s \in \mathcal{S5}$ whether $M_s \vDash \varphi$. The model-checking procedure for $APAL^+$ is a simple modification of the model-checking procedure for $APAL$ of Ågotnes, et al. [1] taking advantage of the characterisation of positive announcements of Lemma 3.5 and Lemma 3.6 as model restrictions that are closed under refinements.

LEMMA 4.1. *Let $M = (S, \sim, V), M' = (S', \sim', V') \in \mathcal{S5}$ be epistemic models. There is a unique, maximal refinement $\mathfrak{R} \subseteq S \times S'$ from M to M' and it is computable in polynomial time.*

This follows from similar reasoning and using a similar algorithm to the analogous result for bisimulations [10], relaxing the **forth** condition appropriately.

THEOREM 4.2. *The model-checking problem for $APAL^+$ is in PSPACE.*

PROOF. (Sketch) We note that PSPACE=APTIME [5]. We adapt the APTIME model-checking procedure for $APAL$ of Ågotnes, et al. [1]. The only modification required is that when we non-deterministically choose a model restriction we must ensure that it corresponds to a positive announcement. From Lemma 3.5 and Lemma 3.6, on finite epistemic models the results of positive announcements correspond exactly to those model restrictions that are closed under refinements. By Lemma 4.1 we can compute the maximal refinement from a model to itself and determine whether a model restriction is closed under refinements in polynomial time. Thus the algorithm remains APTIME. □

We note that if we extend $APAL^+$ with the $\diamondplus^*$ operator for finite sequences of positive announcements, that the model-checking problem is still in APTIME, as on finite models the length of a sequence of non-trivial model restrictions, where each model restriction removes at least one state, is bounded by the number of states in the model.

THEOREM 4.3. *The model-checking problem for $APAL^+$ is PSPACE-hard.*

PROOF. (Sketch) This follows from the same reasoning used by Ågotnes, et al. [1] in the setting of $APAL$, showing that instances of the QBF-SAT problem can be solved through model-checking a $\mathcal{L}_{apal^+}$ formula on an appropriately constructed model. We note that the model is constructed such that each state has a unique valuation, so any restriction of the model is closed under refinements and is definable as the result of a positive announcement (actually a propositional announcement). Therefore the result of any public announcement on this model is also the result of a positive announcement, and the interpretation of $\boxplus$ and $\Box$ is the same on this model. □

5. EXPRESSIVITY

In this section we compare the expressivity of $APAL^+$ to PAL and $APAL$.

If a logic L_1 can express all of the semantic properties that can be expressed in another logic L_2 we say that L_1 is *at least as expressive* as L_2 or equivalently that L_2 is *at most as expressive* as L_1. If two logics L_1 and L_2 are each at least as expressive as the other we say that L_1 is *expressively equivalent* to L_2, and vice-versa. If L_1 is at least as expressive as L_2 and in addition there are semantic properties that can be expressed in L_1 that cannot be expressed in L_2 we say that L_1 is *strictly more expressive* than L_2 or equivalently that L_2 is *strictly less expressive* than L_1. If L_1 and L_2 are neither at least as expressive than the other then we say that L_1 is *incomparable* in expressivity to L_2.

THEOREM 5.1. *$APAL^+$ is expressively equivalent to PAL for a single agent.*

THEOREM 5.2. *$APAL^+$ is strictly more expressive than PAL for multiple agents.*

These results follows from the same reasoning to that used by Balbiani, et al. [3] to show analogous results for $APAL$. It follows that $APAL^+$ is expressively equivalent to $APAL$ for a single agent, however for multiple agents there is a different story.

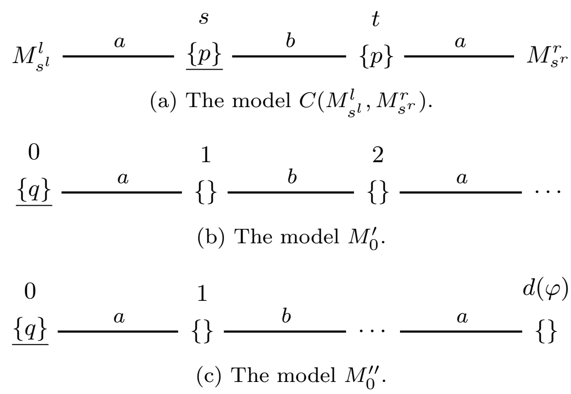
THEOREM 5.3. *$APAL^+$ is not at least as expressive as APAL for multiple agents.*

PROOF. (Sketch) We construct two classes of epistemic models $\mathcal{B}$ and $\overline{\mathcal{B}}$ and show that there is formula in $\mathcal{L}_{apal}$ that can distinguish between the two classes, but there is no formula in $\mathcal{L}_{apal^+}$ with this property. We will use just two agents $a, b \in A$ to construct this class. Let $M^l_{s^l}, M^r_{s^r} \in \mathcal{S5}$ be disjoint epistemic models. We construct a new model $C(M^l_{s^l}, M^r_{s^r}) = (S, \sim, V)$ where:

$$\begin{aligned}
S &= S^l \cup S^r \cup \{s, t\} \\
\sim_a &= \bigcup \left\{ \begin{array}{l} \sim^l_a \setminus \{(s^l, u), (u, s^l) \mid u \in S, u \neq s^l\}, \\ \sim^r_a \setminus \{(s^r, u), (u, s^r) \mid u \in S, u \neq s^r\}, \\ \{(s^l, s), (s, s^l), (s, s), (s^r, t), (t, s^r), (t, t)\} \end{array} \right\} \\
\sim_b &= \sim^l_b \cup \sim^r_b \cup \{(s, s), (s, t), (t, s), (t, t)\} \\
\sim_c &= \sim^l_c \cup \sim^r_c \cup \{(s, s), (t, t)\} \text{ for } c \in A \setminus \{a, b\} \\
V(p) &= V^l(p) \setminus \{s^l\} \cup V^r(p) \setminus \{s^r\} \cup \{s, t\} \\
V(q) &= V^l(q) \cup V^r(q) \text{ for } q \in P \setminus \{p\}
\end{aligned}$$

Let $\mathcal{B}$ be the class of pointed models $C(M^l_{s^l}, M^r_{s^r})_s$ where $M^l_{s^l}$ and $M^r_{s^r}$ are agree on all epistemic formulas, and let $\overline{\mathcal{B}}$ be the class of models $C(M^l_{s^l}, M^r_{s^r})_s$ where $M^l_{s^l}$ and $M^r_{s^r}$ do not agree on all epistemic formulas. Then $\mathcal{B} \models \Box(K_b K_a p \vee K_b \neg K_a p)$. That is, any public announcement that removes s^l must also remove s^r and vice-versa. This is a direct consequence of s^l and s^r agreeing on all epistemic formulas.

Figure 5: Constructions used for Theorem 5.3



(a) The model $C(M^l_{s^l}, M^r_{s^r})$.

(b) The model M'_0.

(c) The model M''_0.

Likewise, $\overline{\mathcal{B}} \not\models \Box(K_b K_a p \vee K_b \neg K_a p)$. That is, there is a public announcement that can remove s^l without removing s^r and vice-versa. This is a direct consequence of s^l and s^r *not* agreeing on all epistemic formulas.

Let $\varphi \in \mathcal{L}_{apal^+}$ be a formula of arbitrary positive announcement logic.

We define the epistemic model $M'_0 = ((S', \sim', V'), 0)$ where:

$$\begin{aligned}
S' &= \mathbb{N} \\
\sim'_a &= \{(n, n), (n, n+1), (n+1, n), (n+1, n+1) \\
&\qquad \mid n \in \mathbb{N}, n \text{ is even}\} \\
\sim'_b &= \{(n, n), (n, n+1), (n+1, n), (n+1, n+1) \\
&\qquad \mid n \in \mathbb{N}, n \text{ is odd}\} \\
V'(q) &= \{0\}
\end{aligned}$$

We also define the epistemic model $M''_0 = M'_0|\{n \in \mathbb{N} \mid n \leq d(\varphi)\}$, the restriction of M to states less than or equal to the modal depth of φ.

We note that $M'_0 \simeq_{d(\varphi)} M''_0$, so the two models agree on all epistemic formulas with modal depth less than or equal to $d(\varphi)$. We also note for every $n \geq m > 0$ that $M'_n \succeq M'_m$ and for every $d(\varphi) \geq n \geq m > 0$ that $M''_n \succeq M''_m$. So for both models the only non-trivial model restrictions that are closed under refinements is the restriction to $\{0\}$, where $M'_0|\{0\} \simeq M''_0|\{0\}$, and the restriction to $\mathbb{N} \setminus \{0\}$ where $M'_n|\mathbb{N} \setminus \{0\} \simeq M''_n|\mathbb{N} \setminus \{0\}$ for every $n \in \mathbb{N}$, so the interpretation of $\diamondplus$ and $\boxplus$ operators must be the same in corresponding states of each model. Therefore both models agree on all arbitrary positive announcement formulas with modal depth less than or equal to $d(\varphi)$.

However the two models do not agree on all epistemic formulas. In M'', every state can reach a state where q is true in at most $d(\varphi)$ steps, but this is not the case for M'.

Thus $C(M'_0, M''_0)_s$ satisfies φ if and only if $C(M'_0, M'_0)_s$ satisfies φ. It follows that φ cannot distinguish $\mathcal{B}$ from $\overline{\mathcal{B}}$ and therefore $APAL^+$ not at least as expressive as $APAL$. $\Box$

We conjecture that $APAL^+$ is incomparable to $APAL$.

6. AXIOMATISATION

In this section we provide a sound and complete axiomatisation for arbitrary positive announcement logic. The axiomatisation is essentially a modified version of the axiomatisation for arbitrary public announcement logic given by Balbiani, et al. [3, 4], but with restrictions to positive announcements in appropriate axioms.

DEFINITION 6.1. *Consider a new symbol* $\sharp$. *The* necessity forms *are defined inductively as:*

$$\psi(\sharp) ::= \sharp \mid (\varphi \to \psi(\sharp)) \mid [\varphi]\psi(\sharp) \mid K_a\psi(\sharp)$$

where $\varphi \in \mathcal{L}_{apal^+}$ *and* $a \in A$.

The possibility forms *are defined inductively as:*

$$\psi(\sharp) ::= \sharp \mid (\varphi \wedge \psi(\sharp)) \mid \langle\varphi\rangle\psi(\sharp) \mid L_a\psi(\sharp)$$

where $\varphi \in \mathcal{L}_{apal^+}$ *and* $a \in A$.

A possibility form is the dual of a necessity form. Necessity and possibility forms contain a unique occurrence of the symbol $\sharp$. If $\psi(\sharp)$ is a necessity or possibility form and $\varphi \in \mathcal{L}_{apal^+}$, then $\psi(\varphi)$ is $\psi(\sharp)[\varphi/\sharp]$ and $\psi(\varphi) \in \mathcal{L}_{apal^+}$.

The axiomatisation APAL^+ is given below.

DEFINITION 6.2. *The axiomatisation* APAL^+ *is a substitution schema consisting of the axioms and rules:*

P	*All propositional tautologies*
K	$\vdash K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$
T	$\vdash K_a\varphi \to \varphi$
4	$\vdash K_a\varphi \to K_aK_a\varphi$
5	$\vdash \neg K_a\varphi \to K_a\neg K_a\varphi$
AP	$\vdash [\varphi]p \leftrightarrow (\varphi \to p)$
AN	$\vdash [\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
AC	$\vdash [\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$
AK	$\vdash [\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
AA	$\vdash [\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$
A+	$\vdash \boxplus\varphi \to [\psi]\varphi$ *where* $\psi \in \mathcal{L}_{el}^+$
MP	*From* $\vdash \varphi$ *and* $\vdash \varphi \to \psi$ *infer* $\vdash \psi$
NecK	*From* $\vdash \varphi$ *infer* $\vdash K_a\varphi$
NecA	*From* $\vdash \varphi$ *infer* $\vdash [\psi]\varphi$
R+$^\omega$	*From* $\vdash \nu([\psi]\varphi)$ *for every* $\psi \in \mathcal{L}_{el}^+$ *infer* $\nu(\boxplus\varphi)$

where $\nu(\sharp)$ *is a necessity form.*

If $\vdash \varphi$ we say that φ is *provable* using the axiomatisation APAL^+. If $\Gamma \subseteq \mathcal{L}_{apal^+}$ is a set of formulas and there are $\gamma_1, \ldots, \gamma_n \in \Gamma$ such that $\vdash (\gamma_1 \wedge \cdots \wedge \gamma_n) \to \varphi$ we say that φ is *deducible* from Γ and we write $\Gamma \vdash \varphi$. If $\Gamma \nvdash \bot$ we say that Γ is *consistent*. If every formula provable using an axiomatisation for a logic is valid in the semantics of the logic we say that the axiomatisation is *sound*. If every formula valid in the semantics of a logic is provable using an axiomatisation for a logic we say that the axiomatisation is *(weakly) complete*. If every set of formulas that is consistent according to an axiomatisation for a logic is satisfiable in the semantics of the logic we say that the axiomatisation is *strongly complete*.

THEOREM 6.3. *The axiomatisation* APAL^+ *is sound and strongly complete for the logic* $APAL^+$.

PROOF. The soundness of the axiomatisation is evident as the axiom **A+** and the rule **R+** follow the semantics of the $\boxplus$ operator, and all remaining axioms and rules are standard from epistemic logic and public announcement logic.

The completeness proof proceeds exactly as in [4], with appropriate restrictions to positive announcements in the cases of **A+** and **R+**. For clarity we show these cases are indeed correct.

The completeness proof is with a standard canonical model technique. The set of states S is defined to be the set of all maximally consistent sets of $\mathcal{L}_{apal^+}$ formulas. The accessibility relations $\sim_a$ refer only to epistemic formulas $K_a\varphi$ and not to the $\boxplus$ operator. To show completeness we must show the Truth Lemma holds: for every maximally consistent set of formulas $s \in S$ and every $\varphi \in \mathcal{L}_{apal^+}$, $\varphi \in s$ if and only if $M_s \models \varphi$. The Truth Lemma is shown by induction on the formula φ. It uses a complexity measure $<_d^{\mathrm{Size}}$ on formulas that we defined identically on $\mathcal{L}_{apal^+}$ as on $\mathcal{L}_{apal}$. The positive arbitrary announcement operator $\boxplus$ only features in the subinductive case $[\psi]\boxplus\chi$ and in the inductive case $\boxplus\psi$. The revised proofs for these cases are as follows. The only change with respect to [4] is the restriction to positive formulas in the appropriate places.

Case $\varphi = [\psi]\boxplus\chi$. The following conditions are equivalent:

1. $[\psi]\boxplus\chi$.
2. For every $\theta \in \mathcal{L}_{el}^+$: $[\psi][\theta]\chi \in s$.
3. For every $\theta \in \mathcal{L}_{el}^+$: $M_s \models [\psi][\theta]\chi$.
4. $M_s \models [\psi]\boxplus\chi$.

From 2 to 1 we use the derivation rule **R+** on the necessity form $[\psi][\theta]\chi$ and the closure of maximally consistent sets under **R+**. From 1 to 2 we use the axiom **A+** and propositional reasoning. From step 2 to 3 we use the complexity measure $<_d^{\mathrm{Size}}$, where we observe that $[\psi]\boxplus\chi$ contains one fewer $\boxplus$ operators than $[\psi][\theta]\chi$, which allows us to use the induction hypothesis. From step 3 to 4 we use the semantics of the $\boxplus$ operator.

Therefore $[\psi]\boxplus\chi \in s$ if and only if $M_s \models [\psi]\boxplus\chi$.

Case $\varphi = \boxplus\psi$. The following conditions are equivalent:

1. $\boxplus\psi \in s$.
2. For every $\theta \in \mathcal{L}_{el}^+$: $[\theta]\psi \in s$.
3. For every $\theta \in \mathcal{L}_{el}^+$: $M_s \models [\theta]\psi$.
4. $M_s \models \boxplus\psi$.

The equivalence between 2 and 3 follows from the fact that for every epistemic formula θ, $[\theta]\psi <_d^{\mathrm{Size}} \boxplus\psi$.

Therefore $\boxplus\psi \in s$ if and only if $M_s \models \boxplus\psi$. $\square$

We note that APAL$^+$ is an infinitary axiomatisation, as the rule $\mathbf{R}+^\omega$ requires an infinite number of premises. As in the axiomatisation of Balbiani, et al. [3] we can simplify this rule to a finitary rule. To do so we first show that if $\langle\!+\!\rangle\varphi$ is satisfiable then so is $\langle p\rangle\varphi$ for some fresh atom p.

Lemma 6.4. *Let $M_s = ((S, \sim, V), s) \in \mathcal{S}5$ be an epistemic model and let $\varphi \in \mathcal{L}_{apal^+}$ be a formula such that $M_s \models \langle\!+\!\rangle\varphi$. Then there exists $M'_{s'} \in \mathcal{S}5$ such that $M'_{s'} \models \langle p\rangle\varphi$ where $p \in P$ does not appear in φ.*

Proof (Sketch). As $M_s \models \langle\!+\!\rangle\varphi$ there exists $\psi \in \mathcal{L}^+_{el}$ such that $M_s \models \langle\psi\rangle\varphi$. Let $Q = \{q_n \mid n \in \mathbb{N}\} \subseteq P$ be an infinite set of atoms not appearing in φ or ψ. We define $M' = (S', \sim', V')$ where:

$$\begin{aligned} S' &= \{\top, \bot\} \times S \\ \sim'_a &= \{((x, s), (y, t)) \mid x, y \in \{\top, \bot\}, s, t \in S, s \sim_a t\} \\ V'(q_0) &= \{\top\} \times [\![\psi]\!]_M \\ V'(q_{n+1}) &= \{\top, \bot\} \times V(q_n) \text{ for every } n \in \mathbb{N} \\ V'(p) &= \{\top, \bot\} \times V(p) \text{ for every } p \in P \setminus Q \end{aligned}$$

We claim that $M'_{(\top,s)} \models \langle q_0\rangle\varphi$. This follows from the fact that the model restriction $M'_{(\top,s)}|q_0$ is isomorphic to the model restriction $M_s|q_0$, but with the atoms in Q renamed and a new atom q_0 introduced that is equivalent to $\top$. As the atoms in Q do not appear in φ then the only effect that changing these variables can have in the interpretation of φ is in the positive announcements that are considered in interpreting the $\langle\!+\!\rangle$ and $\boxplus$ operators of φ. We note that any positive announcement available in $M'_{(\top,s)}|q_0$ has an equivalent positive announcement available in $M_s|q_0$, and vice-versa, found by renaming the variables in Q. $\Box$

Lemma 6.4 can be generalised to possibility forms:

Lemma 6.5. *Let $M_s = ((S, \sim, V), s) \in \mathcal{S}5$ be an epistemic model, let $\varphi \in \mathcal{L}_{apal^+}$, and let $\psi(\sharp)$ be a possibility form such that $M_s \models \psi(\langle\!+\!\rangle\varphi)$. Then there exists $M'_{s'} \in \mathcal{S}5$ such that $M'_{s'} \models \langle p\rangle\psi(\langle\!+\!\rangle\varphi)$ where $p \in P$ does not appear in $\psi(\langle\!+\!\rangle\varphi)$.*

As in the axiomatisation of *APAL* by Balbiani, et al. [3] this allows us to form a sound and weakly complete *finitary* axiomatisation by replacing the infinitary rule $\mathbf{R}+^\omega$ in APAL$^+$ with a finitary alternative of the form:

$$\mathbf{R}+^\mathbf{1} \quad \text{From } \nu([p]\varphi) \text{ infer } \nu(\boxplus\varphi)$$

where $\nu(\sharp)$ is a necessity form and p is a fresh atom.

The soundness of $\mathbf{R}+^\mathbf{1}$ follows from Lemma 6.5, and the weak completeness of the resulting finitary axiomatisation follows from the same reasoning as in Balbiani, et al. [3]. As the resulting axiomatisation is a finitary axiomatisation, it is recursively enumerable.

7. FUTURE WORK

We have yet to show whether the satisfiability problem for $APAL^+$ is decidable. The satisfiability problem for $APAL^+$ is to determine for a given formula whether the formula is satisfiable. As we have given a sound and complete axiomatisation this is equivalent to determining for a given formula whether the formula is provable using the axiomatisation.

While we have shown that $APAL^+$ is not at least as expressive as $APAL$, we do not know whether it is strictly less expressive than $APAL$ or it is incomparable to $APAL$. We also have yet to consider in depth the addition of the $\langle\!+\!\rangle^*$ operator for finite sequences of positive announcements to the logic, and particularly, how the operator effects the expressivity, decidability and complexity of the logic.

We conjecture that the satisfiability problem for $APAL^+$ is decidable and that $APAL^+$ is incomparable in expressivity to $APAL$. The main justification for these conjectures is related to the reason that the proof of undecidability for $APAL$ [8] would not work directly for $APAL^+$. The proof that $APAL$ is undecidable relies on the ability to express as a $\mathcal{L}_{apal}$ formula that two states of an epistemic model agree on all epistemic formulas. This is possible essentially because the $\Box$ operator quantifies over all epistemic formulas. In $APAL^+$ however the $\boxplus$ operator is only able to quantify over positive formulas, enough to express that a state agrees with all of the positive formulas of another state, but not enough to express agreement on all epistemic formulas. Conversely the $\Box$ operator of $APAL$ appears to quantify over too many formulas to express that a state agrees with all of the positive formulas of another state, and so we conjecture that there are $\mathcal{L}_{apal^+}$ formulas that do not have equivalent $\mathcal{L}_{apal}$ formulas and the two logics are incomparable in expressivity.

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